

Washington State

Essential Academic Learning Requirements: Mathematics & Strands and Targets for the Washington Assessment of Student Learning in Mathematics

Introduction to Mathematics

Mathematics for Today and Tomorrow

The use of mathematics continues to grow at a rapid rate, spreading into new fields and with ever-new applications. Several factors—growth of technology, increased applications, impact of computers, and expansion of mathematics itself—have combined in the past century to extend greatly both the scope and the application of the mathematical sciences. The changes must be reflected in the schools if our students are to be well prepared for tomorrow's world.

What is Mathematics?

Mathematics is a language and science of patterns. As a language of patterns, mathematics is a means for describing and creating the world in which we live. In its symbols and vocabulary, the language of mathematics is a universal means of communication about relationships and patterns.

Measurement, geometric sense, and statistics are ways in which mathematics is used to describe the world in which we live. Measurement and geometric sense are also creative tools. Algebraic sense and probability are used to describe events and to extend beyond what is known to build models and make predictions. Measurement, computation, statistics, and probability are also used for decision making. At the base of all mathematics is our number system and the ways we use our number sense to understand and represent mathematical ideas and information.

In all of these uses of mathematical language and tools, humans collect, organize, and represent mathematical information – which requires mathematical communication skills. Decision making, predicting, and creating all require problem-solving skills. Fundamental to mathematical communication and problem solving are mathematical reasoning skills.

More than at any other time in history, society is placing demands on citizens to interpret and use mathematics to make sense of information and complex situations. This requires making connections between mathematical concepts and between mathematics and the world beyond school. Computers and other technologies have increased our capacities for dealing with numbers for collecting, organizing, representing, and analyzing data. Tables, lists of numbers, graphs of data, and statistics summarizing information occur in every form of the media.

To be well informed as adults and to have access to desirable jobs, students today require an education in mathematics that goes far beyond what was needed by students in the past. All students must develop and sharpen their skills; deepen their understanding of mathematical concepts and processes; and hone their problem-solving, reasoning, and communication abilities while using mathematics to make sense of and to solve compelling problems. All students need a deep understanding of mathematics. For this to occur, rigorous mathematical content must be reorganized, taught, and assessed in a problem-solving environment. For students to develop this deeper level of understanding, their knowledge must be connected to a variety of ideas and skills across topic areas and grade levels in mathematics to other subjects taught in school as well as to situations outside the classroom.

Given these many tasks, stages, and purposes of mathematics, the Essential Academic Learning Requirements (EALRs) are organized around five essentials. The first is focused on the basic concepts and procedures of mathematics. The second is focused on the processes used to solve mathematical problems. The third is focused on the thinking skills needed to solve problems. The fourth is focused on mathematical communication skills. The fifth is focused on the critical importance of making connections within mathematics and between mathematics and the world beyond school. In the pages that follow, these five essential requirements of mathematics are further defined.

Essential Academic Learning Requirements— Mathematics

1. The student understands and applies the concepts and procedures of mathematics.

To meet this standard, the student will:

- 1.1. Understand and apply concepts and procedures from number sense—number and numeration; computation; and estimation.
- 1.2. Understand and apply concepts and procedures from measurement—attributes, units, and systems; procedures and estimation.
- 1.3. Understand and apply concepts and procedures from geometric sense—properties and relationships; locations and transformations.
- 1.4. Understand and apply concepts and procedures from probability and statistics—outcomes and probabilities; data collections; central tendencies; data representation and interpretation.
- 1.5. Understand and apply concepts and procedures from algebraic sense—patterns and functions; symbols and notations; evaluation solving.

2. The student uses mathematics to define and solve problems.

To meet this standard, the student will:

- 2.1. Define problems—identify known, missing, or extraneous information.
- 2.2. Construct solutions—organize the necessary information, use appropriate strategies and procedures, and determine a solution that is viable and mathematically correct.

3. The student uses mathematical reasoning.

To meet this standard, the student will:

- 3.1. Analyze information—interpret and compare mathematical information.
- 3.2. Conclude—draw and support conclusions and evaluate procedures.
- 3.3. Verify results—justify results, check for reasonableness of results, and validate thinking.

4. The student communicates knowledge and understanding in both everyday and mathematical language.

To meet this standard, the student will:

- 4.1. Gather information—extract mathematical information by reading, listening, and observing.
- 4.2. Organize, represent, and share information—express, explain, or represent mathematical ideas using language and notation appropriately for audience and purpose.

5. The student understands how mathematical ideas connect within mathematics, to other subject areas, and to real-world situations.

To meet this standard, the student will:

- 5.1. Relate concepts and procedures within mathematics—use conceptual and procedural understandings among content strands and use equivalent models and representations.
- 5.2. Relate mathematical concepts and procedures to other disciplines—identify and use mathematical patterns, thinking, and modeling in other subject areas.
- 5.3. Relate mathematical concepts and procedures to real-world situations—understand the connections between mathematics and problem-solving skills used every day at work and at home.

HIGH SCHOOL MATHEMATICS

ASSESSMENT STRANDS AND TARGETS

The Essential Academic Learning Requirements (EALRs) have been further defined as strands and targets for the purposes of assessment. The mathematics strands and targets are given below.

Content strands refer to traditional definitions of content areas in mathematics (number sense, measurement, geometric sense, statistics and probability, algebraic sense). Process strands refer to the EALRs for mathematical communication, problem solving, mathematical reasoning, and making connections within mathematics. Learning targets further detail the knowledge and skills items are designed to assess within each strand. The learning targets for each grade level, were derived from the benchmark indicators¹ in the EALR Technical Document that was published in 1997. In the pages that follow, the numbers in parentheses after each learning target refer to particular indicators.

Each WASL mathematics item is designed to assess students' attainment of one target within a strand. Strand reports on WASL score reports are based on total item scores within a particular strand.

¹ Benchmark indicators are the detailed learning expectations defined for grades 4, 7, and 10 in the EALR Technical Document.

CONTENT STRANDS AND LEARNING TARGETS

Strand 1: Number Sense (NS)

NS01 (Number and Numeration) Demonstrate understanding of the concepts and symbolic representations of rational numbers including whole number powers, square roots of perfect squares, and numbers written in scientific notation; demonstrate understanding of the relative values of rational numbers including whole number powers and square roots of perfect squares; demonstrate understanding of and use the distributive property and properties of addition and multiplication with rational numbers including integers (1.1.1, 1.1.2, 1.1.3)

NS02 (Ratio and Proportion) Demonstrate understanding of and apply the concepts of ratio, percent, and both direct and inverse proportion (1.1.4)

NS03 (Conceptual Understanding of Operations) Demonstrate understanding of the meaning of operations with rational numbers including whole number powers and square roots (1.1.5)

NS04 (Computation) Complete multi-step computations with combinations of rational numbers including integers, whole number powers, and square roots of perfect squares, using order of operations (1.1.6)

NS05 (Estimation) Identify when an approximation is appropriate; use estimation to determine the reasonableness of answers in situations involving multi-step computations with rational numbers including integers, whole number powers, and square roots (1.1.8)

Strand 2: Measurement (ME)

ME01 (Attributes and Dimensions) Demonstrate understanding of how a change in one linear dimension affects surface area and volume or how changes in two linear dimensions affect perimeter, area, and volume (1.2.1)

ME02 (Units and Systems) Demonstrate understanding of rate and other derived units of measurement; demonstrate understanding of how to convert within the U.S. or metric system to achieve an appropriate level of precision; explain why different situations require different levels of precision (1.2.2, 1.2.3)

ME03 (Procedures) Use formulas, including the Pythagorean Theorem, to determine measurements of triangles, prisms, or cylinders (1.2.5)

ME04 (Estimated Measurements) Identify situations in which estimated measurements are sufficient; use estimation to obtain reasonable measurements at an appropriate level of precision (1.2.6)

Strand 3: Geometric Sense (GS)

GS01 (Properties and Relationships) Demonstrate understanding of the characteristics of cylinders, cones, and pyramids and the relationships among 1-dimensional, 2-dimensional, and 3-dimensional figures; draw, describe, and/or compare 1-dimensional, 2-dimensional, and 3-dimensional shapes and figures, including prisms, cylinders, cones, and pyramids; use the Pythagorean Theorem to determine if a triangle is a right triangle (1.3.1, 1.3.2)

GS02 (Locations and Transformations) Use geometric properties to describe or identify the location of points on coordinate grids; use multiple transformations including translations, reflections, and/or rotations to create congruent figures in any or all of the four quadrants (1.3.3, 1.3.4)

Strand 4: Probability and Statistics (PS)

PS01 (Probability) Demonstrate understanding of the concepts of compound, dependent and independent events; determine and use probabilities of compound, dependent, and independent events (1.4.1, 1.4.2)

PS02 (Data Collection and Central Tendencies) Identify possible sources of bias in questions, data collection methods, samples, and/or measures of central tendency in a situation and describe how such bias can be controlled; identify clusters and outliers and determine how they may affect measures of central tendency (1.4.3, 1.4.4)

PS03 (Data Representation and Interpretation) Draw a reasonable line to describe the data represented by a scatter plot and determine whether a straight line is an appropriate way to describe the trend in the data; read and interpret data presented in tables of ordered pairs and scatter plots and make predictions based on the given data; use statistics to support different points of view or evaluate a statistical argument based on data (1.4.5, 1.4.6)

Strand 5: Algebraic Sense (AS)

AS01 (Patterns and Functions) Recognize, extend, or create a pattern or sequence of pairs of numbers representing a linear function; identify or write a rule to describe a pattern, sequence, and/or a linear function (1.5.1, 1.5.2)

AS02 (Symbols and Notations) Represent relationships between quantities using squares, cubes, and square roots; use variables to write expressions, linear equations, and inequalities that represent situations involving rational numbers, whole number powers, and square and cube roots (1.5.3, 1.5.4)

AS03 (Evaluating and Solving) Simplify expressions; solve multi-step equations, systems of equations, and one-step inequalities (1.5.5, 1.5.6)

III. PROCESS STRANDS AND LEARNING TARGETS

Strand 6: Solves Problems Reasons Logically (SR)

SR01 (Define Problems) Identify questions to be answered in complex situations; recognize when information is missing or extraneous; identify what is known and unknown in complex situations (2.1.1, 2.1.2, 2.1.3)

SR02 (Construct Solutions) Select and organize relevant information; use appropriate concepts and procedures from number sense, measurement, geometric sense, probability and statistics, and algebraic sense; use a variety of strategies and approaches; determine whether a solution is viable, mathematically correct, and answers the question(s) asked (2.2.1, 2.2.2, 2.2.3, 2.2.4)

SR03 (Analyze Information) Interpret, compare, and integrate mathematical information from multiple sources (3.1.1)

SR04 (Conclude) Draw conclusions and support them using inductive and deductive reasoning; evaluate procedures and make needed revisions (3.2.1, 3.2.2)

SR05 (Construct Solutions and Verify Results) Use viable strategies and appropriate concepts and procedures to construct a solution; justify results using inductive and deductive reasoning; check for reasonableness of results; validate thinking and mathematical ideas using models, known facts, patterns, relationships, counterexamples, and/or proportional reasoning (2.2.2, 2.2.3, 3.3.1, 3.3.2, 3.3.3)

Strand 7: Communicates Understanding (CU)

CU01 (Gather Information) Develop or select an efficient system for collecting mathematical information for a given purpose; extract mathematical information for a given purpose from multiple sources using reading and observation (4.1.1, 4.1.2)

CU02 (Organize, Represent and Share Information) Organize, clarify, and refine mathematical information for a given purpose; use everyday and mathematical language and notation in appropriate and efficient forms to clearly express or represent complex ideas and information; explain and/or represent complex mathematical ideas and information in ways appropriate for audience and purpose in a context that is relevant to tenth grade students (4.2.1, 4.2.2, 4.2.3)

Strand 8: Makes Connections (MC)

MC01 (Connections within Mathematics) Use concepts and procedures from multiple mathematics content strands in a given problem or situation; relate and use different mathematical models and representations of the same situation (5.1.1, 5.1.2)